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A SIMPLIFIED FORMULA FOR THE CHANGE IN ORDER OF INTERFERENCE DUE TO CHANGES IN TEMPERATURE AND PRESSURE OF AIR

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A change in the density of the air between two interferometer mirrors, as in the Fabry-Perot interferometer or the Fizeau apparatus for expansion coefficients, by altering the refractive index, causes a change in the optical difference of path and in the order of interference. The change in density may be occasioned by either a change in the temperature or in the pressure, and the relation holding between the refractive index (μ) and the density (d) is expressed by Gladstone and Dale's law:

$$\frac{\mu - 1}{d} = \text{constant}.$$

In the interference method for expansion coefficients a correction must therefore be applied to the observed change in the order of interference, to take account of any change in the density of the air. The following formula for this correction was published by Pulfrich in 1893:[1]

$$K = l(t_2 - t_1)\frac{b_1}{760}\cdot\frac{1}{1+\alpha t_1}\cdot\frac{1}{1+\alpha t_2}\left[\frac{2(\mu-1)\alpha}{\lambda}\right]$$
$$-l(b_2 - b_1)\cdot\frac{1}{1+\alpha t_2}\left[\frac{2(\mu-1)}{760\lambda}\right]$$

Where:

l = distance between mirrors.
t_1 = lower temperature (in degrees centigrade).
t_2 = upper temperature.
b_1 = pressure in mm of mercury at temperature t_1.
b_2 = pressure in mm of mercury at temperature t_2.
α = expansion coefficient of air.
μ = refractive index at 0°, 760.
λ = wave length.

[1] Zs. für Instrk., **13**, p. 456. The sign of the expression in the original paper is reversed if the expression is to represent the correction, not the error.

The above formula has been used by Reimerdes, Randall, and Minchin.

Some months ago I derived independently the following much simpler formula for the same correction:

$$K=\left(\frac{b_1}{1+\alpha t_1}-\frac{b_2}{1+\alpha t_2}\right)l\left[\frac{\mu-1}{380\lambda}\right]$$

After using this formula at the Bureau of Standards for several months, I noticed that the Pulfrich formula could be reduced to it by simple algebraic transformations.

The part of the expression in square brackets is constant for constant wave length. The value of $\frac{1}{1+\alpha t}$ may be obtained from the Landolt-Börnstein tables or from a graph constructed for the temperatures most frequently used. Assuming the use of the same reference tables in each case, and that the constant parts of each expression have been evaluated, the operations required to obtain a numerical value are as follows:

	Subtractions	Multiplications	Divisions
Pulfrich form	3	8	1
New form	1	4	0

The following is the derivation of the new formula:

In addition to above symbols, let μ_1, μ_2; n_1, n_2; d_1, d_2 represent refractive indices, orders of interference, and densities of air at temperatures t_1 and t_2, respectively. Let λ be the wave length in vacuo.

Then

$$n_1=\frac{2\mu_1 l}{\lambda} \qquad (1)$$

$$n_2=\frac{2\mu_2 l}{\lambda} \qquad (2)$$

$$n_2-n_1=\frac{2\,l(\mu_2-\mu_1)}{\lambda} \qquad (3)$$

By Gladstone and Dale's law

$$\mu_2 = 1 + c\,d_2 \tag{4}$$

$$\mu_1 = 1 + c\,d_1 \tag{5}$$

Where c = Gladstone and Dale constant. But by the simple gas laws,

$$d_2 = \frac{d_0 b_2}{(1+\alpha t_2)\ 760} \tag{6}$$

and

$$d_1 = \frac{d_0 b_1}{(1+\alpha t_1)\ 760} \tag{7}$$

Where d_0 = density of air at 0° and 760 mm. Representing c by $\frac{\mu - 1}{d_0}$, substituting (7) and (6) in (5) and (4); and (5) and (4) in (3), and simplifying we find

$$n_2 - n_1 = \frac{l(\mu - 1)}{380\lambda}\left(\frac{b_2}{1+\alpha t_2} - \frac{b_1}{1+\alpha t_1}\right)$$

This is the change in the order of interference as the temperature increases from t_1 to t_2 and the pressure changes from b_1 to b_2 (increase or decrease). The correction is the above expression with sign reversed, thus:

$$K = \left(\frac{b_1}{1+\alpha t_1} - \frac{b_2}{1+\alpha t_2}\right) l \left[\frac{\mu - 1}{380\lambda}\right]$$

WASHINGTON, October, 1912.